APPROXIMATE CALCULATION OF PRESSURE DROP IN LAMINAR FLOW OF GENERALIZED NEWTONIAN LIQUID THROUGH CHANNEL OF NONCIRCULAR CROSS SECTION

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The paper presents a modification of the equations of Rabinowitsch–Mooney type for an approximate calculation of pressure drop in laminar flow of generalized Newtonian liquid through a straight channel whose cross section forms a simple continuous area. The suitability of the suggested procedure of calculation of pressure drop is demonstrated by the comparison of calculation results with both the published and original results of numerical solution and experiments.

A basic task of chemical engineering in the field of flow of liquid through a closed channel consists in the determination of dependence of pressure drop of liquid upon its volume flow rate and rheological properties.

A survey of possible calculation procedures of pressure drop in laminar flow of non-compressible liquids through channels of noncircular cross section is given e.g. by Bukovsky[1]. For a chemical-engineering calculation it is sufficient to adopt approximate methods which either are based on a generalization of the relationships derived for the flow through channels of a simple cross section (see e.g. refs[2,3]) or replace the flow of a rheologically more complex liquid through a channel of given geometry by a flow of a rheologically simpler liquid[4].

The calculation procedure suggested by us can be classified as a method of the first type mentioned above. It consists in a modification of the Rabinowitsch–Mooney equation in a way analogous to the procedure used by us[5 – 7] in the calculation of pressure drop in laminar flow of Generalized Newtonian Liquid (GNL) through an immobile random layer of particles. The procedure is based on a modification of the characteristic linear dimension of system.

THEORETICAL

A laminar flow of GNF through a pipe of circular cross section and a planar slot can be described by an equation of Rabinowitsch–Mooney type in the form

$$D_\mathrm{w} \;\equiv\; (3-\Omega)\,u_\mathrm{ch}/l_\mathrm{ch} \;=\; \left[(3+\Omega)/\tau_\mathrm{w}^{(2+\Omega)}\right]\int_0^{\tau_\mathrm{w}} \tau^{(1+\Omega)}\,D(\tau)\,\mathrm{d}\tau\,, \tag{1}$$

where the shape characteristic of channel is Ω = 1 and 0 for the pipe and slot, respectively.

In Eq. (*1*) the term D_w stands for a kinematic consistency variable,

$$\tau_\mathrm{w} \;=\; \Delta p\; l_\mathrm{ch}/L \tag{2}$$

is a dynamic consistency variable with the meaning of shear stress at the wall of channel,

$$u_\mathrm{ch} \;=\; \dot{V}/S \tag{3}$$

is the characteristic (mean) flow rate, l_ch is the characteristic linear dimension of channel which in this case is identical with the hydraulic radius of channel

$$l_\mathrm{ch} \;\equiv\; r_\mathrm{h} \;=\; S/O\,, \tag{4}$$

and finally, $D(\tau)$ is the dependence of rate of shear deformation D upon the shear stress τ, which is generally given by the flow curve of GNF or the respective flow model.

For a Newtonian liquid ($D(\tau) = \tau/\mu$, where μ is the dynamic viscosity), Eq. (*1*) after integration and modification assumes the form

$$\Delta p\; l_\mathrm{ch}^2/(L\,u_\mathrm{ch}\;\mu) \;=\; 3\;-\;\Omega\,, \tag{5}$$

whereas for a power-law liquid ($D(\tau) = (\tau/K)^{1/n}$, where K and n are parameters of the model), the respective form is:

$$\Delta p\; l_\mathrm{ch}^{n+1}/(K\,L\,u_\mathrm{ch}^n) \;=\; [(3-\Omega)\,(2+\Omega+1/n)/(3+\Omega)]^n\,. \tag{6}$$

The equation (*1*) can easily be integrated also for other flow models expressing explicitly the dependence of rate of deformation D upon the shear stress τ. However, for a determination of pressure drop of a GNF flowing through channels it usually is not necessary to use flow models which are more complex than the power-law model if the *K* and *n* parameters are determined from rheometric data in the corresponding interval of deformation rates[3,7].

We expect that Eq. (*1*) and Eqs (*5*) and (*6*) derived therefrom, which at $\Omega = 1$ or 0 and $l_{ch} = r_h$ are exactly valid for a laminar flow of GNF through a pipe of circular cross section or planar slot, will be approximately valid also for a flow through a straight channel of any noncircular cross section representing a simple continuous area.

For the channels of the given geometry we presume that the shape characteristic Ω of the channel will have a value from the interval of $0 < \Omega < 1$. The characteristic linear dimension l_{ch} of channel will be connected with the hydraulic radius r_h by the relationship:

$$l_{ch} = \beta r_h , \tag{7}$$

where $\beta > 0$ is a characteristic of the channel whose value is considered independent of the flow properties of GNF.

The quantity Ω, which is independent of the flow properties of GNF, can be introduced, e.g., in the following form:

$$\Omega = (r_h/r_p)^m , \tag{8}$$

where

$$r_p = (S/4\pi)^{0.5} \tag{9}$$

is the cross-sectional radius of channel, which corresponds to the hydraulic radius of a pipe with circular cross section of the same area *S* as that of the channel considered. Equation (*8*) gives the required values of $\Omega = 1$ and 0 for the pipe of circular cross section ($r_h = r_p$) and the planar slot ($r_p \to \infty$), respectively.

The exponent *m* in Eq. (*8*) may be submitted to optimization considerations. However, it turns out that it is sufficient simply to choose $m = 2$, which has the consequence that the quantities l_{ch}, r_h, and r_p in modified Eq. (*5*) have the same exponent.

The characteristic linear dimension l_{ch} for the considered channel of noncircular cross section is then determined from the known solution for Newtonian liquid from Eq. (*5*), where Ω is given by Eq. (*8*) with the value of $m = 2$. The magnitude of β characteristic can be determined with the help of Eq. (*7*).

RESULTS AND DISCUSSION

In order to verify the suggested way of calculation of pressure drop, we have used the results of our own numerical solution of laminar flow of power-law liquid through a straight channel of semicircular cross section[8] as well as those given in literature for the flow of power-law liquid through a channel of rectangular cross section[9 – 11], channels of symmetrical L- and U-shaped cross sections[1], elliptical cross section[12], and a cross section in the shape of equilateral triangle[10,11].

The agreement between the pressure drop values Δp_{calc} calculated from Eq. (*6*) and the Δp values determined for identical flow conditions by the above-mentioned authors was evaluated by means of the magnitude of the mean deviation

$$\delta = [(1/N) \sum_{i=1}^{N} \delta_i^2]^{1/2} , \qquad (10)$$

where δ_i is the relative percent deviation given by Eq. (*11*).

$$\delta_i = (\Delta p/\Delta p_{\mathrm{calc.}} - 1)\ 100 . \qquad (11)$$

At the same time, the Δp values were compared also with the data of pressure drop Δp_{calc} calculated from Eq. (*12*) suggested by Miller[3].

$$\Delta p\, r_{\mathrm{n}}^{n+1}/(K\, L\, u_{\mathrm{ch}}^{n}) = [3n + 1/(2n)]^n\, (\lambda/16)^n \qquad (12)$$

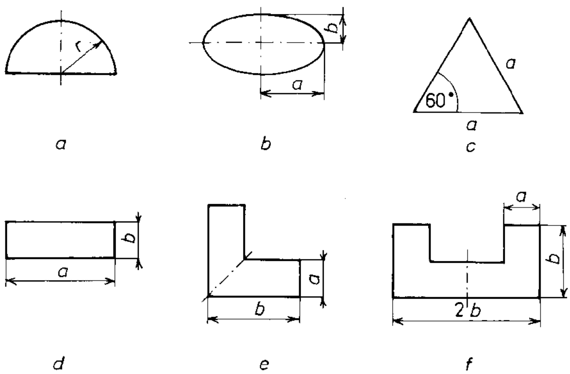

FIG. 1
Shapes of cross sections of tested channels. *a* semicircle, *b* ellipse, *c* equilateral triangle, *d* rectangle, *e* symmetrical L-profile, *f* symmetrical U-profile

TABLE I
Deviations δ_i and δ of numerical calculation[8] of pressure drop for channel of semicircular cross section (Fig. 1*a*) and data calculated from Eqs (*6*) and (*12*)

Characteristics		*n*	δ_i, %	
β	Ω		Eq. (*6*)	Eq. (*12*)
1.07	0.747	0.9	–0.1	–0.6
		0.8	–0.1	–1.1
		0.7	–0.1	–1.6
		0.6	0.0	–2.2
		0.5	0.3	–2.3
$N = 5$		δ, %	0.2	1.7

TABLE II
Deviations δ_i and δ of numerical calculation[8] of pressure drop for channels of rectangular cross section (Fig. 1*d*) and data calculated from Eqs (*6*) and (*12*)

Characteristics			*n*	δ_i, %	
a/*b*	β	Ω		Eq. (*6*)	Eq. (*12*)
1.00	1.12	0.785	0.75	0.7	–1.8
			0.50	1.0	–3.9
			0.33	0.8	–5.7
			0.20	–1.4	–9.2
0.75	1.11	0.769	0.75	0.5	–1.8
			0.50	0.4	–4.2
			0.33	0.8	–5.4
			0.20	–1.6	–8.9
0.50	1.09	0.698	0.75	–0.1	–1.7
			0.50	–0.6	–3.9
			0.33	–1.1	–5.7
0.20	1.04	0.436	0.75	0.4	0.4
			0.50	–2.2	–2.3
			0.33	–4.3	–4.8
	$N = 14$		δ, %	1.5	5.0

In this relationship, λ means the shape characteristic of channel whose value is determined (analogously to β) from the known solution of flow of Newtonian liquid with application of Eq. (*12*) for $n = 1$.

Furthermore, the comparison of Eqs (*5*), (*7*) and Eq. (*12*) for $n = 1$ gives:

$$(3 - \Omega)/\beta^2 \quad = \quad \lambda/8 \; . \qquad (13)$$

TABLE III

Deviations δ_i and δ of experimental data[10,11] of pressure drop for channels of rectangular cross section (Fig. 1*d*) and data calculated from Eqs (*6*) and (*12*)

Author	Characteristics			n	δ_i, %	
	a/b	β	Ω		Eq. (*6*)	Eq. (*12*)
10	1.00	1.12	0.785	0.915	7.8	7.5
				0.890	6.1	5.3
				0.740	8.8	6.4
				0.575	4.9	5.6
				0.405	6.4	1.0
11				0.650	3.7	2.7
				0.530	3.5	3.2
10	0.50	1.09	0.698	0.740	2.5	4.3
				0.610	3.3	2.0
				0.405	2.3	1.5
11	0.33	1.06	0.589	0.650	5.1	1.9
				0.530	1.9	6.0
11	0.20	1.04	0.436	0.650	1.5	3.9
	$N = 13$			δ, %	5.0	4.4

TABLE IV
Deviations δ_i and δ of numerical calculation[1] of pressure drop for channels of symmetrical L-profile (Fig. 1*e*) and data calculated from Eqs (*6*) and (*12*)

Characteristics			n	δ_i, %	
a/b	β	Ω		Eq. (*6*)	Eq. (*12*)
1.0	1.12	0.785	0.750	0.7	–1.7
			0.500	1.0	–4.0
			0.377	1.1	–5.0
0.9	1.13	0.777	0.750	0.7	–2.3
			0.500	1.1	–4.4
			0.377	1.2	–5.8
0.8	1.14	0.754	0.750	0.7	–2.6
			0.500	1.1	–4.9
			0.377	1.2	–6.3
0.7	1.15	0.715	0.750	0.5	–2.7
			0.500	1.1	–5.0
			0.377	1.2	–6.3
0.6	1.13	0.660	0.750	–0.1	–2.6
			0.500	0.0	–5.0
			0.377	–0.1	–6.4
0.5	1.11	0.589	0.750	–0.9	–2.7
			0.500	–0.7	–4.4
			0.377	0.1	–6.4
0.4	1.08	0.503	0.750	–0.7	–1.8
			0.500	–1.6	–3.8
			0.377	–2.7	–5.8
0.3	1.06	0.401	0.750	–0.9	–1.5
			0.500	–1.8	–2.7
			0.377	–3.8	–5.1
	N = 24		δ, %	1.3	4.4

The obtained values of deviations δ_i and δ are summarized in Tables I – IX for channels of the cross sections tested whose shapes are given in Fig. 1. The tables also give the corresponding values of β and Ω characteristics.

The tabulated deviation values δ_i and δ show that the way suggested for calculation of pressure drop of GNF flowing through a straight channel whose cross section forms a simple continuous area gives very good results similar to those of Miller's method. With application of Eq. (*6*), the individual deviations δ_i vary within the interval from –5.8% to 8.8%, and with Eq. (*12*) from –9.2% to 7.5%. The mean deviations δ do not exceed 5.0% for both the equations.

In the majority of the cases tested (75%) the calculation by Eq. (*6*) gives smaller deviations δ_i than that by Miler's equation (*12*). The reason lies obviously in the fact that Eq. (*6*) applies exactly to both the limiting geometries considered for the cross section of channel (circular and slot), whereas Eq. (*12*) applies exactly to a pipe only.

The calculation of pressure drop according to Eq. (*6*) gives more favourable values if compared with the results of numerical calculations (Tables I, II, IV, VI, VIII) than if compared with experimental data (Tables III, V, VII, IX), which can be rationalized by

TABLE V
Deviations δ_i and δ of experimental data[1] of pressure drop for channels of symmetrical L-profile (Fig. 1*e*) and data calculated from Eqs (*6*) and (*12*)

Characteristics			n	δ_i, %	
a/b	β	Ω		Eq. (*6*)	Eq. (*12*)
1.0	1.12	0.785	0.868	–0.7	–4.8
			0.381	1.7	–1.6
0.7	1.15	0.715	0.868	–2.9	–7.1
			0.381	2.1	–2.1
0.5	1.11	0.590	0.868	–3.2	–4.5
			0.381	1.8	–3.1
0.3	1.06	0.401	0.868	–7.5	–4.1
			0.381	4.1	–1.3
	N = 8		δ, %	3.6	4.0

TABLE VI
Deviations δ_i and δ of numerical calculation[1] of pressure drop for channels of symmetrical U-profile (Fig. 1*f*) and data calculated from Eqs (*6*) and (*12*)

Characteristics			n	δ_i, %	
a/b	β	Ω		Eq. (*6*)	Eq. (*12*)
1.0	1.09	0.698	0.750	0.2	–1.7
			0.500	–0.3	–3.9
			0.377	–0.8	–5.2
0.9	1.11	0.647	0.750	0.0	–2.3
			0.500	–0.1	–4.6
			0.377	–0.5	–6.0
0.8	1.11	0.589	0.750	–0.5	–2.5
			0.500	–0.8	–4.9
			0.377	–0.7	–6.1
0.7	1.10	0.525	0.750	–0.7	–2.2
			0.500	–1.5	–4.8
			0.377	–2.1	–6.4
0.6	1.09	0.467	0.750	–0.5	–1.6
			0.500	–2.2	–4.7
			0.377	–3.0	–6.3
0.5	1.07	0.385	0.750	–0.1	–0.9
			0.500	–1.6	–3.2
			0.377	–4.2	–6.4
0.4	1.05	0.311	0.750	–0.3	–0.1
			0.500	–2.0	–2.2
			0.377	–5.0	–5.0
0.3	1.03	0.234	0.750	–0.3	0.5
			0.500	–2.0	–2.2
			0.377	–5.8	–5.2
	N = 24		δ, %	2.1	4.1

TABLE VII
Deviations δ_i and δ of experimental data[1] of pressure drop for channels of symmetrical U-profile (Fig. 1*f*) and data calculated from Eqs (*6*) and (*12*)

Characteristics			n	δ_i, %	
a/b	β	Ω		Eq. (*6*)	Eq. (*12*)
0.8	1.11	0.589	0.868	0.7	–0.6
			0.381	1.0	–4.5
0.7	1.10	0.525	0.868	–1.2	–2.0
			0.381	1.4	–3.4
0.5	1.07	0.385	0.868	0.5	–0.2
			0.381	–2.1	–5.4
0.3	1.03	0.234	0.868	–1.4	–0.9
			0.381	–5.7	–4.2
	$N = 8$		δ, %	2.3	3.2

TABLE VIII
Deviations δ_i and δ of numerical calculation[12] of pressure drop for channels of elliptical cross section (Fig. 1*b*) and data calculated from Eqs (*6*) and (*12*)

Characteristics			n	δ_i, %	
a/b	β	Ω		Eq. (*6*)	Eq. (*12*)
0.75	1.00	0.970	0.8	–0.1	–0.1
			0.6	–0.1	–0.1
			0.4	0.0	–0.1
0.50	1.01	0.843	0.8	–0.8	–0.7
			0.6	–0.6	–0.5
			0.4	–0.2	–0.2
0.25	1.04	0.546	0.8	1.6	1.5
			0.6	2.2	1.8
			0.4	3.3	2.5
	$N = 9$		δ, %	1.5	1.2

the fact that the experimental data are loaded with greater errors, viz. up to ±4% (refs[1,10]).

The tabulated δ_i and δ data also show a tendency (if the results of the approximate and numerical calculations of pressure drop are compared) to mildly grow with decreasing flow index *n* of the liquid, especially so with the results obtained from Eq. (*12*). When applying the comparison with experimental data (Tables III, V, VII, IX) this tendency disappears due obviously to the above-mentioned experimental error.

Tables I – IX also present the values of channel characteristic β whose availability is necessary for predictions of pressure drop according to Eq. (*6*). They are in the interval of $1 < \beta \leq 1.20$ for the cross section shapes tested. The corresponding λ values (Eq. (*13*)) then lie in the interval of $13.3 \leq \lambda \leq 20.9$.

CONCLUSIONS

An equation of the Rabinowitsch–Mooney type has been suggested for approximate calculations of pressure drop in laminar flow of power-law liquid through a straight closed channel whose cross section forms a simple continuous area.

The suitability of its application was demonstrated by our own results as well as by results taken from literature for channels of semicircular, rectangular, and elliptical cross sections and cross sections of the shapes of equilateral triangle, symmetrical L- and U-profiles. Our present research work is focused on an extension of the procedure to the calculation of pressure drop in channels with built-in structures.

TABLE IX
Deviations δ_i and δ of experimental data[10,11] of pressure drop for a channel of equilateral-triangle cross section (Fig. 1*c*) and data calculated from Eqs (*6*) and (*12*)

Author	Characteristics		*n*	δ_i, %	
	β	Ω		Eq. (*6*)	Eq. (*12*)
10	1.20	0.605	0.890	–0.6	–2.3
			0.640	–2.0	–7.4
11			0.845	1.3	–1.1
			0.670	6.7	–0.3
	N = 4		δ, %	3.6	3.9

SYMBOLS

a, b	dimensions characterizing cross section of channel, m
D	rate of shear deformation, s^{-1}
D_w	consistency variable, s^{-1}
K	parameter of power-law model, Pa s^n
L	length of channel, m
l_{ch}	characteristic linear dimension of system, m
N	number of experiments
n	parameter of power-law model, dimensionless
O	perimeter of channel, m
Δp	pressure drop, Pa
r	radius, m
r_h	hydraulic radius, m
r_p	cross-sectional radius, m
S	cross section of channel, m^2
u_{ch}	characteristic rate of system, m s^{-1}
$\dot{V}$	volume flow rate, m^3 s^{-1}
β	characteristic of channel, dimensionless
Ω	shape characteristic of channel, dimensionless
δ	mean deviation, %
δ_i	relative deviation, %
λ	characteristic of channel according to Miller[3]
μ	dynamic viscosity, Pa s
τ	tangential stress, Pa
τ_w	consistency variable, Pa

Indexes

calc	calculated from Eq. (*6*) or Eq. (*12*)

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